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Fyss300/2 Filters

Supervisor:

Abstract

In this experiment we studied three different kind of filters: RC, LC and RLC and compared the theoretical attenuation and phase shift with the measured ones by plotting the results with gnuplot software. For RC-filter at low frequencies the measured attenuation followed the theoretical ones, but after cutoff it did't decrease as fast as the theoretical. The theoretical and measured phase shift of RC-filter followed each other only at high frequencies and in LC-case they followed each other only at low frequencies. For LC-filter and RLC-filter the theoretical resonance frequency were higher than the measured one, which may be due to some disturbance affected by the coil in circuit. In both of LC and RLC case the measured attenuation plottings followed well the theoretical one, but the measured peak was shorter. The measured and theoretical Q-value of RLC-filter were about the same, but the theoretical was a bit larger, which means the peak of theoretical plotting should be higher than measured, which is true according the the measurements.

Contents

1 Introduction

Filters are circuits, which are used to modify signal.[4, p. 507] There is two main type of filters: low pass filters and high pass filters.[2, p. 24] Low pass filter passes low frequencies and blocks high frequencies and the high pass filter passes high frequencies and blocks low ones.

Filters may be constructed from resistors, capacitor and coils. In this excercise we study RC-filter, LC-filter and RLC-filter by inspecting the attenuation and phase shift of those filters. Attenuation means the amplitude loss, when signal is gone through the filter. The phase shift of filter describes the distance of output and input signal measured in the same phase.

2 Theoretical background

When studuing filters there is input voltage V_{in} and output voltage V_{out} of the form

$$V_{in} = |V_{in}| \sin(\omega t)$$
$$V_{out} = |V_{out}| \sin(\omega t + \phi),$$

where ω is frequency, t is time and ϕ is phase shift caused by filters.

2.0.1 Impedances

The impedances for resistor, capacitor and inductor are

$$\begin{aligned} Z_R &= R \\ Z_C &= \frac{-i}{\omega C} \\ Z_L &= i\omega L. \end{aligned} \tag{1}$$

For ideal inductor the impedance has only the imaginary component expressed above. But in real case it has also an inner resistance R_L.

2.0.2 Attenuation

The proportion of frequency to amplitude is usually represented as logarithmic scale. The attenuation λ describes the ratio of signal amplitude and reference signal amplitude and it can be expressed as desibels

$$\lambda = 10 \log_{10} \frac{A_1^2}{A_0^2} = 20 \log_{10} \frac{A_1}{A_0}.,\tag{2}$$

where the amplitude A can be voltage amplitude V. When considering the relation of input and output voltages we can use impedances of the components in filter circuit

$$\frac{V_{out}}{V_{in}} = \frac{Z_{out}}{Z_{in}}.\tag{3}$$

2.0.3 Phase shift

The measured phase shift of two signal can be defined by multiplying the time difference by frequency

$$\phi = \Delta t \cdot \omega = 2\pi \Delta t \cdot f. \tag{4}$$

That describes the interval between input and output signal, which are in the same phase. To define the theoretical value for phase shift one has to transform the output and input voltage propotion into form, which includes real part a and imaginary part b as the equation below:

$$\frac{V_{out}}{V_{in}} = \frac{Z_{out}}{Z_{in}} = a + ib. \tag{5}$$

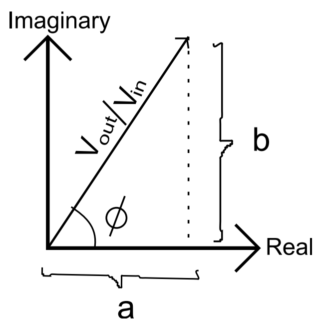


Figure 1: The real part a and an imaginary part b of the equation 5 create a right-angled triangle, where the angle ϕ is the phase shift

The phase shift is described visually in the figure 1, where the imaginary b and real part a of equation 5 are shown. Thus the tangent of the phase shift ϕ can be defined using the right-angled triangle of the figure

$$\tan\phi = \frac{b}{a} \Rightarrow \phi = \arctan\left(\frac{b}{a}\right). \tag{6}$$

2.1 RC-filters

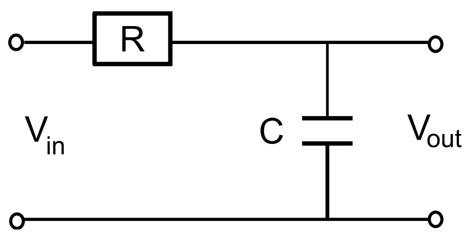


Figure 2: RC-filter

The RC-filter circuit used in this experiment is presented in the figure 2. There is one resistor and one capacitor in RC-filter, so calculating the relation of input and output voltages we need the impedances of resistor and capacitor represented in the equation 1. Let's define the relation of input and output signal of RC-filter using equation 3:

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= \frac{Z_C}{Z_R + Z_C} = \frac{\frac{-i}{\omega C}}{R + \frac{-i}{\omega C}} \\ &= \frac{-i}{R\omega C - i} \cdot \left(\frac{R\omega C + i}{R\omega C + i}\right) \\ &= \frac{-iR\omega C + 1}{(R\omega C)^2 + 1},\end{aligned}$$

where i is imaginary unit, R resistance and C capacitance. Now we can transform this to the form as the equation 5 by writing down the real and imaginary part separately

$$\frac{V_{out}}{V_{in}} = \frac{1}{(R\omega C)^2 + 1} + i \cdot \frac{-R\omega C}{(R\omega C)^2 + 1}.$$

The phase shift can be calculated using the equation 6 with values $a = \frac{1}{(R\omega C)^2+1}$ and $b = \frac{-R\omega C}{(R\omega C)^2+1}$:

$$\phi = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{\frac{-R\omega C}{(R\omega C)^2+1}}{\frac{1}{(R\omega C)^2+1}}\right) = \arctan\left(-R\omega C\right). \tag{7}$$

The relation of V_{out} and V_{in} can be expressed as

$$\begin{aligned}\left|\frac{V_{out}}{V_{in}}\right| &= \sqrt{(a)^2 + (b)^2} = \sqrt{\left(\frac{1}{(R\omega C)^2 + 1}\right)^2 + \left(\frac{-R\omega C}{(R\omega C)^2 + 1}\right)^2} \\ &= \sqrt{\frac{(R\omega C)^2 + 1}{((R\omega C)^2 + 1)^2}} = \frac{1}{\sqrt{(R\omega C)^2 + 1}},\end{aligned} \tag{8}$$

which can be used in the equation 2 to obtain the attenuation.

2.2 LC-filters

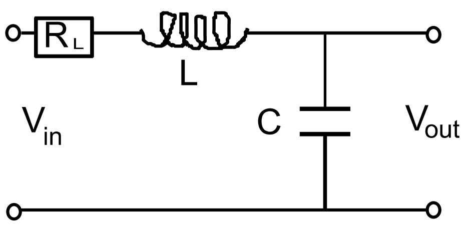


Figure 3: LC-filter

LC-filter is very similar to RC-filter, but the resistor is replaced with an inductor. As shown in figure 3, the inductor has inductance L and resistance R_L. For LC-filter the proportion of input and output signal amplitudes is

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= \frac{Z_C}{R_L + Z_L + Z_C} = \frac{\frac{-i}{\omega C}}{R_L + i\omega L + \frac{-i}{\omega C}} = \frac{1}{1 - \omega^2 CL + i\omega CR_L} \\ &= \frac{1}{(1 - \omega^2 CL) + i\omega CR_L} \cdot \frac{(1 - \omega^2 CL) - i\omega CR_L}{(1 - \omega^2 CL) - i\omega CR_L} \\ &= \frac{(1 - \omega^2 CL) - i\omega CR_L}{(1 - \omega^2 CL)^2 + (\omega CR_L)^2}.\end{aligned}$$

This can be expressed as the equation 5, where $a = \frac{(1-\omega^2 CL)}{(1-\omega^2 CL)^2+(\omega C R_L)^2}$ and $b = \frac{-\omega C R_L}{(1-\omega^2 CL)^2+(\omega C R_L)^2}$. Thus the phase shift is

$$\phi = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{\left(\frac{-\omega C R_L}{(1-\omega^2 CL)^2+(\omega C R_L)^2}\right)}{\left(\frac{(1-\omega^2 CL)}{(1-\omega^2 CL)^2+(\omega C R_L)^2}\right)}\right) = \arctan\left(\frac{-\omega R_L C}{1-\omega^2 LC}\right). \tag{9}$$

The magnitude of input and output voltage relation is

$$\begin{aligned} \left|\frac{V_{out}}{V_{in}}\right| &= \sqrt{(a)^2+(b)^2} \\ &= \sqrt{\left(\frac{(1-\omega^2 CL)}{(1-\omega^2 CL)^2+(\omega C R_L)^2}\right)^2 + \left(\frac{-\omega C R_L}{(1-\omega^2 CL)^2+(\omega C R_L)^2}\right)^2} \\ &= \sqrt{\frac{(1-\omega^2 CL)^2+(\omega C R_L)^2}{\left((1-\omega^2 CL)^2+(\omega C R_L)^2\right)^2}} = \frac{1}{\sqrt{(1-\omega^2 CL)^2+(\omega C R_L)^2}}. \end{aligned} \tag{10}$$

The resonance frequency occurs, when impedance between input voltage and output is at minimum. Thus

$$\begin{aligned} Z = Z_C + Z_L + R_L &= i\left(\frac{-1}{\omega C} + \omega L\right) \\ \Rightarrow \quad |Z| &= \sqrt{(R_L)^2 + \left(\frac{-1}{\omega C} + \omega L\right)^2} \\ \Rightarrow \quad |Z|_{minimum} &= \sqrt{R_L^2} \\ \Rightarrow \quad \frac{-1}{\omega C} + \omega L = 0 \quad &\Rightarrow \quad \omega = \frac{1}{\sqrt{LC}}. \end{aligned}$$

Now we got the theoretical resonance frequency, which need to be transformed

$$\omega_{RT} = \frac{1}{\sqrt{LC}} = 2\pi f_{RT} \quad \Rightarrow \quad f_{RT} = \frac{1}{2\pi\sqrt{LC}}. \tag{11}$$

2.3 RLC-filters

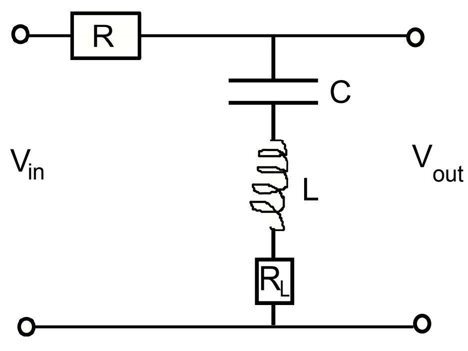


Figure 4: RLC-filter

RLC-filter is represented in picture 4. There is showed also a resistance of the inductor R_L, but it's very small and I neglect in during calculations. Now the output voltage and input voltage relation can be expressed as

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= \frac{Z_C + Z_L}{Z_C + Z_L + R} = \frac{\frac{-i}{\omega C} + i\omega L}{\frac{-i}{\omega C} + i\omega L + R} \\ &= \frac{1-\omega^2 CL}{1-\omega^2 CL + i\omega CR} \cdot \frac{(1-\omega^2 CL) - i\omega CR}{(1-\omega^2 CL) - i\omega CR} \\ &= \frac{(1-\omega^2 CL)((1-\omega^2 CL) - i\omega CR)}{(1-\omega^2 CL)^2 + (\omega CR)^2},\end{aligned}$$

where we can clearly see the real part $a = \frac{(1-\omega^2 CL)^2}{(1-\omega^2 CL)^2+(\omega CR)^2}$ and imaginary part $b = \frac{-(1-\omega^2 CL)(\omega CR)}{(1-\omega^2 CL)^2+(\omega CR)^2}$. Using those terms a and b we got the phase shift equation

$$\phi = \arctan\Big(\frac{b}{a}\Big) = \arctan\Big(\frac{-(1-\omega^2 CL)(\omega CR)}{(1-\omega^2 CL)^2}\Big) = \arctan\Big(\frac{-\omega CR}{1-\omega^2 CL}\Big). \tag{12}$$

Let's next figure out the magnitude equation for output and input voltage relation

$$\begin{aligned}\Big|\frac{V_{out}}{V_{in}}\Big| = \sqrt{a^2+b^2} &= \sqrt{\Big(\frac{(1-\omega^2 CL)^2}{(1-\omega^2 CL)^2+(\omega CR)^2}\Big)^2 + \Big(\frac{-(1-\omega^2 CL)(\omega CR)}{(1-\omega^2 CL)^2+(\omega CR)^2}\Big)^2} \\ &= \sqrt{\frac{(1-\omega^2 CL)^2\Big((1-\omega^2 CL)^2+(\omega CR)^2\Big)}{\Big((1-\omega^2 CL)^2+(\omega CR)^2\Big)^2}} \\ &= \frac{1-\omega^2 CL}{\sqrt{(1-\omega^2 CL)^2+(\omega CR)^2}}.\end{aligned} \tag{13}$$

The resonance frequency occurs, when impedance between output and input is at minimum. It can be calculated at the same way than in LC-filter case. Thus

$$\begin{aligned}Z = Z_C + Z_L + R = R + i(\frac{-1}{\omega C} + \omega L) \\ |Z| = \sqrt{(R)^2 + \Big(\frac{-1}{\omega C} + \omega L\Big)^2},\end{aligned}$$

which is at minimum, when

$$\frac{-1}{\omega C} = \omega L \quad \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

And when $\omega_{RT} = 2\pi f_{RT}$ we get the resonance frequency

$$f_{RT} = \frac{1}{2\pi\sqrt{LC}}. \tag{14}$$

The Q-factor describes the quality of the circuit. In this case it tells how well our RLC-filter stores energy.[3, p. 5] It can be expressed as

$$Q = \frac{\omega_R}{\Delta\omega} = \frac{\omega_R L}{R}, \tag{15}$$

where ω_R is resonance frequency and $\Delta\omega$ is bandwith of the signal wave. Thus the wider bandwith the smaller Q-value.

3 Experimental methods

This laboratory study took two days to be performed completely. At the first day we made measurings for RC-filters and LC-filters and at second day we performed RLC-filter measurings.

The resistance of resistor in RC-filter, capacitance of capacitor and inductance of inductor were measured with Megger B131 multimeter. The inductor resistance was measured with Finest multimeter and resistor of RLC-filter was measured with UNIT-T multimeter (UT58B).

3.1 RC-filter

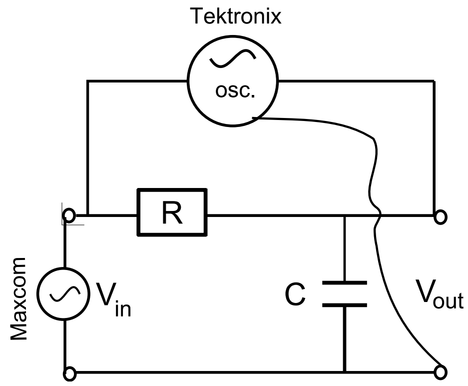


Figure 5: Measurement settings for RC-filter

The supervisor set up the coupling for RC-filter using Bimboard connection plate, wires, a resistor and a capacitor. First we checked the signal of Maxcom power supply (MX-9300) connected only with a digital oscilloscope Tektronix (TDS3012) and then we tested the filter coupling with oscilliscope.

Before the actual measurements we coupled the RC-filter with the oscilloscope and power supply as shown in the figure 5, where the channel 1 was coupled with input voltage and channel 2 was coupled with output voltage. The frequency of power supply were adjusted by turning a button. We started at low frequency values and wrote down the values of input voltage, output voltage and phase shift time. The voltages were displayed on the oscilloscope screen as a height of the signal figure and the phase shift time were defined as an horizontal distance of input and output voltages figures. We picked up those values using several differ frequency and rose the frequency value after every measurings.

3.2 LC-filter

After RC-filter measurement we just changed the resistance component to inductor and didn't do anything else modification to the circuit. The setup for LC-filter is shown in picture 6, where R_L is the inner resistance of the inductor.

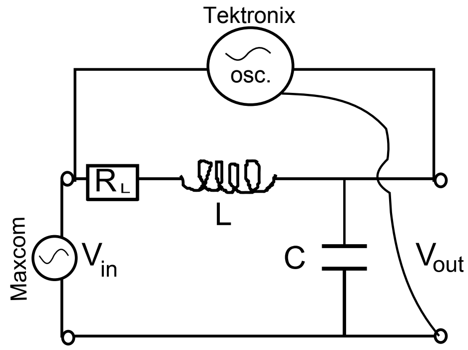


Figure 6: Measure settings for LC-filter

The measurement was performed quite similarly than in RC-filter case. First we adjusted the small frequency and then we rose it writing down the input voltage, output voltage and phase shift values for every chose frequency.

3.3 RLC-filter

The RLC-filter setup in shown in the figure 7, where capacitor and inductor are the same as in earlier measurement, but in this case we used about $1k\Omega$ resistance. The measurement method was the same as in the RC-filter and LC-filter; we rose the frequency and picked the values of input voltage, output voltage and phase shift time.

But the output voltage wasn't a symmetric sine wave, when the frequency was near the resonance frequance. The wave figure had two marked maximum points and looked deformed. But when frequency was far enough from the resonance, the signal seemed to be non-disturbed sine wave.

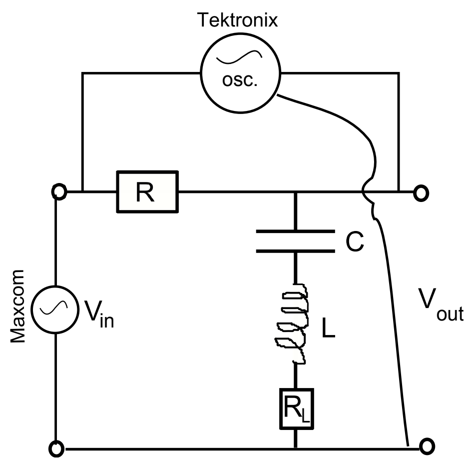


Figure 7: Measure settings for RLC-filter

4 Results

The measured values for components are listed below:

- Resistor (RC): $R = 6670\Omega$, $\delta R = 0.005R + 3\Omega \approx 40\Omega$
- Capacitor: $C = 22.18 \cdot 10^{-9}$F, $\delta C = 0.01C + 0.05 \cdot 10^{-9}F\approx 0.3 \cdot 10^{-9}$F
- Inductor: $L = 0.010$H, $\delta L = 0.02L + 0.005$H≈ 0.006H
- Inductor: $R_L = 10\Omega$
- Resistor (RLC): $R = 987\Omega$

The errors are defined using the user manual of Megger B131 multimeter [1] and they are rounded up.

Before plotting and calculations I created tables as text files from the results of the measurement log in attachment 1. Using those tables as text files I generated with gnuplot 4.4 software the calculations and plottings. The gnuplot logs are in the attachment 2 and the tables of measurement results in attachment 3. The plottings of the attenuations are performed at log-log scale, where x-axis (frequency) is allways of the form $\log_{10}(f)$ and y-axis is $\lambda = 20\log_{10}(A_{out}/A_{in})$ (attenuation). When plotting the phase shift the x-axis is again logarithmic $\log_{10}(f)$, but y-axis is ϕ (degrees).

4.1 RC-filter

4.1.1 Attenuation

The theoretical attenuation λ_T of RC-filter can be calculated with the equation 2 using the equation 8 thus

$$\lambda_T = 20\log_{10}\Big(\frac{1}{\sqrt{(\omega RC)^2+1}}\Big) = 20\log_{10}\Big(\frac{1}{\sqrt{(2\pi f RC)^2+1}}\Big),$$

where $\omega = 2\pi f$ is measured frequency, R is measured resistance and C measured capacitance. Using that equation I calculated and plotted with gnuplot the theoretical attenuation values for every measured frequency in figure 8(a).

The measured attenuation can be calculated using the measured input V_{in} and output voltage V_{out} values in the measurement log using the attenuation equation 2

$$\lambda_M = 20\log_{10}(\frac{V_{out}}{V_{in}}).$$

Also those values were calculated and plotted with gnuplot and it's represented in the figure 8(b). Both the theoretical and measured attenuation values are plotted in the figure 9, where the square symbols are the theoretical and crosses are the measured values. In this case the attenuation decreases, when frequency is increaseing, which means the output voltage decreases approaching zero.

I made a liner fitting to the measured attenuation plotting to show the slope of the area, where the attenuation is decreasing. The fitting is the straight line wrote in the figure 8(b) and I setted its' range to be from 3 to 5. The fitting function was of the form $f(x) = a + b \cdot x$, where the gnuplot generated the values $a = 21 \pm 3$ and $b = -8 \pm 1$. The slope of the fitting is actually the parameter $b = \frac{\Delta \lambda_M}{\Delta \log_{10}(f)} = -8 \pm 1$.

The cutoff frequency occurs, when $\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{2}}$. The theoretical value for RC-filter cutoff frequency can be calculated with equation $\omega_{cT} = \frac{1}{RC} = 2\pi f_{cT} \Rightarrow f_{cT} = \frac{1}{2\pi RC} =$ so

$$f_{cT} = \frac{1}{2\pi 6670\Omega \cdot 22.18 \cdot 10^{-9}\text{F}} \approx 1075.80\text{Hz}.$$

The error for this can be calculated using the propagation of error method $\delta f = \sqrt{\sum_i (\frac{\partial f}{\partial x_i}\delta x_i)^2}$, thus

$$\begin{aligned} \delta f_{cT} &= \frac{1}{2\pi}\sqrt{\Big(\frac{\partial \omega_{cT}}{\partial R}\delta R\Big)^2 + \Big(\frac{\partial \omega_{cT}}{\partial C}\delta C\Big)^2} = \frac{1}{2\pi}\sqrt{\Big(\frac{-C\delta R}{(RC)^2}\Big)^2 + \Big(\frac{-R\delta C}{(RC)^2}\Big)^2} \\ &= \frac{1}{2\pi}\sqrt{\Big(\frac{-22.18 \cdot 10^{-9} \cdot 40}{(6670 \cdot 22.18 \cdot 10^{-9})^2}\Big)^2 + \Big(\frac{-6670 \cdot 0.3 \cdot 10^{-9}}{(6670 \cdot 22.18 \cdot 10^{-9})^2}\Big)^2} \approx 20\text{Hz}. \end{aligned} \tag{16}$$

Now the theoretical cutoff frequency is $f_{cT} = (1080 \pm 20)$Hz

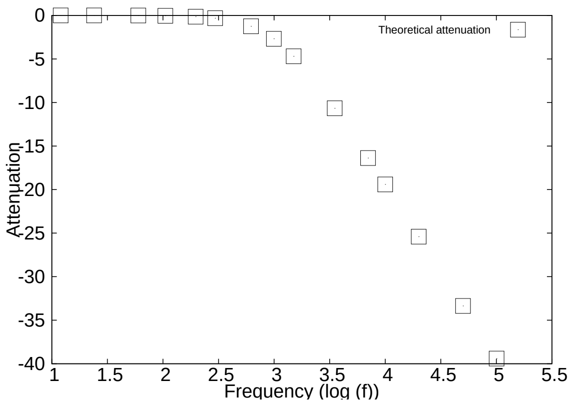
The measured value for cutoff frequency can be figured out using the linear fitting equation

$$\begin{aligned} f(x) = \lambda_M = 20\log_{10}(\frac{V_{out}}{V_{in}}) &= 20\log_{10}(\frac{1}{\sqrt{2}}) = 21 - 8x \\ \Rightarrow x &= \frac{21 - 20\log_{10}(\frac{1}{\sqrt{2}})}{8}, \end{aligned}$$

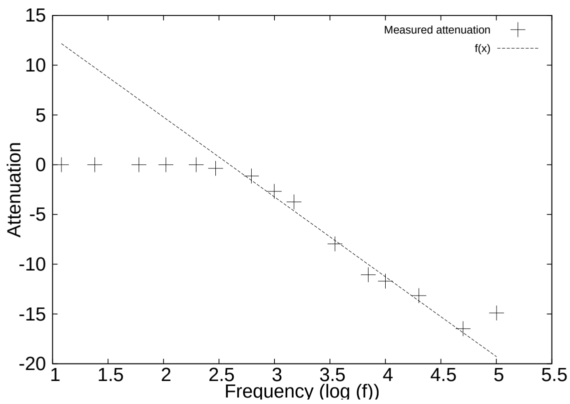
where x is actually the x-axis of the figure, which is defined to be $\log_{10}(f_{cM})$, thus we obtain the measured cutoff frequency f_{cM} expressing

$$\begin{aligned} x = \log_{10}(f_{cM}) &= \frac{21 - 20\log_{10}(\frac{1}{\sqrt{2}})}{8} \\ \Rightarrow f_{cM} &= 10^{\frac{21-20\log_{10}(\frac{1}{\sqrt{2}})}{8}} \approx 1003\text{Hz}, \end{aligned}$$

which is smaller than theoretical value. Even if we take the error of theoretical value into account they don't be equal.



(a) Theoretical attenuation



(b) Measured attenuation

Figure 8: Theoretical and measured attenuation plots of RC-filter

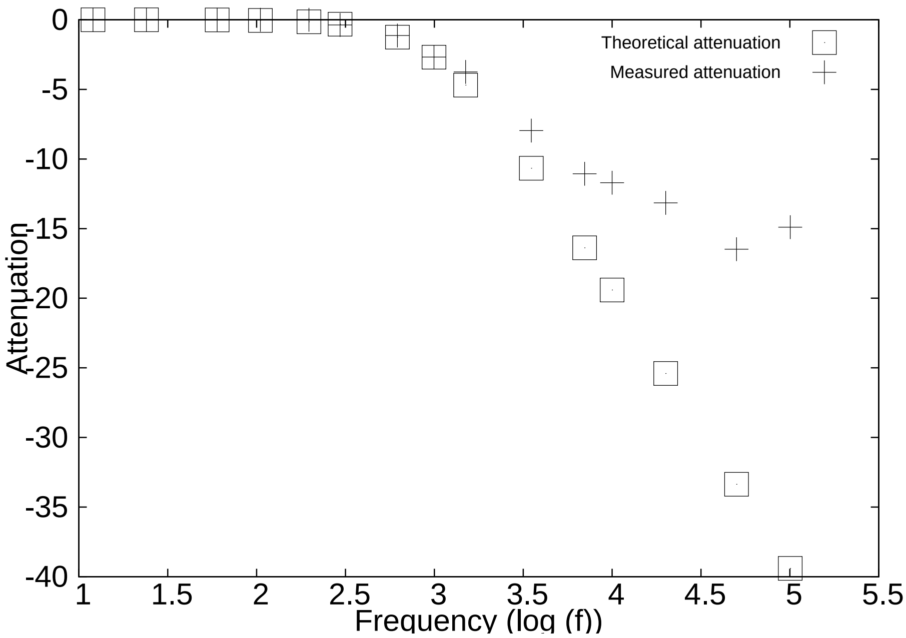


Figure 9: Theoretical and measured attenuations of RC-filter

4.1.2 Phase shift

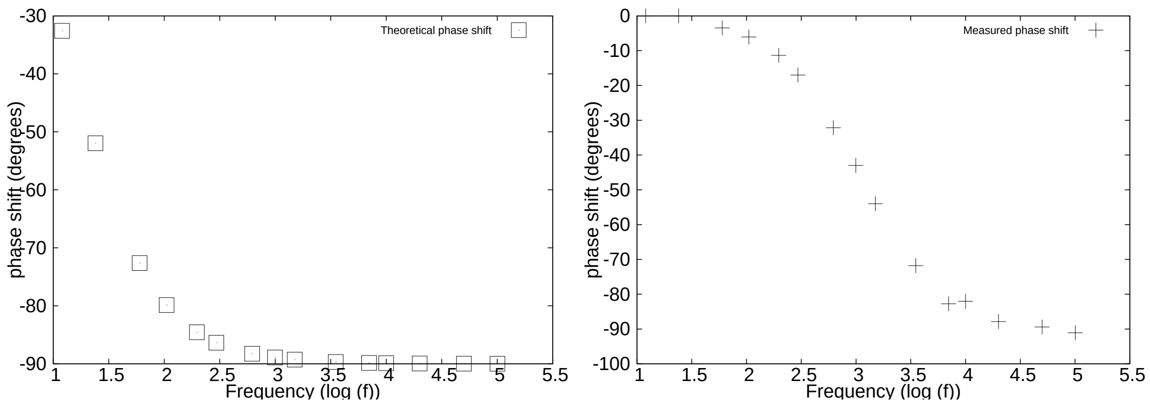
Let's next figure out the theoretical phase shift using the equation 7

$$\phi_T = \arctan(-R\omega C) = \arctan(-2\pi fRC) = \arctan(-360fRC).$$

And the measured phase shift is defined as the equation 4

$$\phi_M = -2\pi\Delta tf = -360\Delta tf.$$

I made the plottings for theoretical (figure 10(a)) and measured (figure 10(b)) phase shift. Additionally both of the phase shifts are represented in the same figure 11. That figure shows the measured phase shift didn't give any value in lower frequencies, which is due to the fact the measured phase shift time was zero, which affects the equation comes zero.



(a) Theoretical phase shift (b) Measured phase shift

Figure 10: Theoretical and measured phase shift plottings for RC-filter

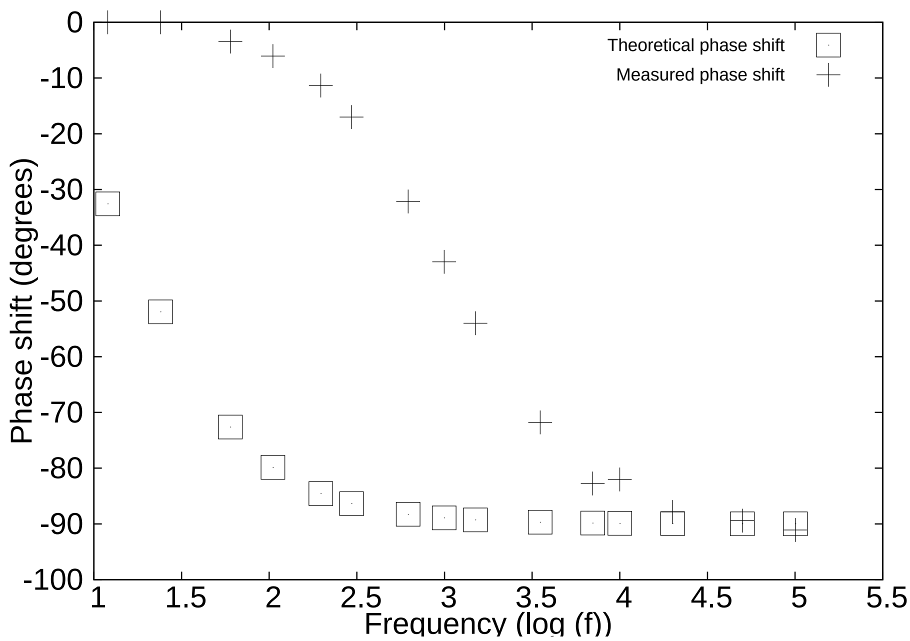


Figure 11: Theoretical and measured attenuations of RC-filter

4.2 LC-filter

4.2.1 Attenuation

The theoretical attenuation value is defined as the equation 2, where the amplitude relation is defined to be the equation 10. Thus the theoretical attenuation is

$$\lambda_T = 20\log_{10}\Big(\frac{1}{\sqrt{(1-\omega^2 CL)^2 + (\omega C R_L)^2}}\Big),$$

where C is measured capacitance, L measured inductance R_L is the measured resistance of inductor and $\omega = 2\pi f$ frequency. The theoretical attenuation for every measured frequency is plotted in the figure 12(a).

The measured attenuation is defined the same way than at the RC-filter case, thus

$$\lambda_M = 20\log_{10}\Big(\frac{V_{out}}{V_{in}}\Big),$$

where V_{out} is output voltage and V_{in} input voltage of LC-filter marked in the attachment 1. Plottings for those measured attenuation values are shown in the figure 12(b), where is also a liner fitting for the last part of the plottings. I setted the linear fit range from 4.5 to 5.5, which includes the plottings values after the peak. Linear fitting function was of the form $f(x) = a + b \cdot x$, where $a \approx 7.1$ and $b \approx -8.4$ generated by gnuplot. Now the slope of attenuation is $b = \frac{\Delta\lambda_M}{\Delta\log(f)} = -8.4$.

The resonance frequency f_R at the measured plottings occurs, when impedance between input voltage and output voltage is at minimum. The theoretical value $f_{RT} = \frac{1}{2\pi}\omega_{RT}$ for this can be calculated using the equation 11

$$f_{RT} = \frac{1}{2\pi\sqrt{LC}} \approx 10686.6\text{Hz},$$

where $L = 0.010\text{H}$ and $C = 22.18 \cdot 10^{-9}\text{F}$. Let's calculate also the error with propagation of error method

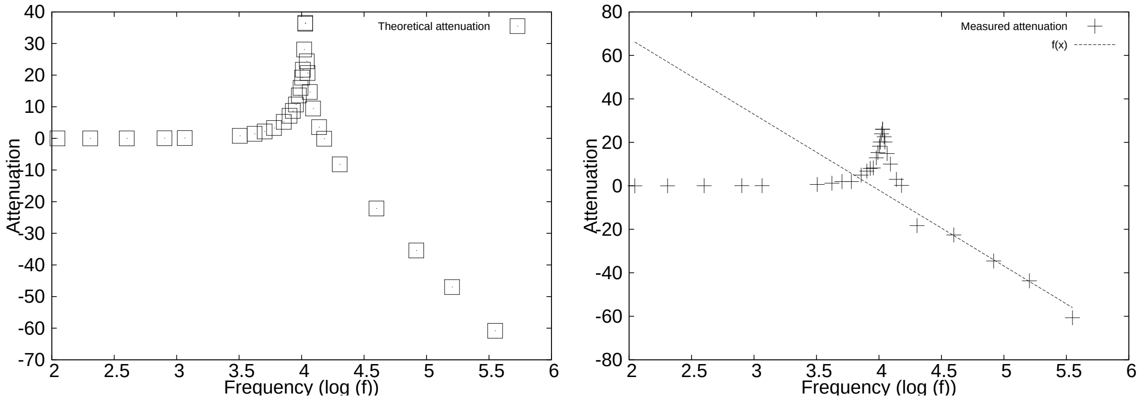
$$\delta f_{RT} = \frac{1}{2\pi}\sqrt{\Big(\frac{\partial\omega_{RT}}{\partial L}\delta L\Big)^2 + \Big(\frac{\partial\omega_{RT}}{\partial C}\delta C\Big)^2} = \frac{1}{2\pi}\sqrt{\Big(\frac{\frac{1}{2}C\delta L}{(LC)^{3/2}}\Big)^2 + \Big(\frac{\frac{1}{2}L\delta C}{(LC)^{3/2}}\Big)^2}$$
$$= \sqrt{\Big(\frac{\frac{1}{2}22.18\cdot 10^{-9}\cdot 0.006}{(0.010\cdot 22.18\cdot 10^{-9})^{3/2}}\Big)^2 + \Big(\frac{\frac{1}{2}0.010\cdot 0.3\cdot 10^{-9}}{(0.010\cdot 22.18\cdot 10^{-9})^{3/2}}\Big)^2} \approx 4000\text{Hz}.$$

So the theoretical resonance frequency is $f_{RT} = (11000 \pm 4000)\text{Hz}$. As one can note the error is a quite large, which is due to the error of inductor showed in the begin of the section 4.

From graph the resonance frequency can be defined to occur, when attenuation is as high as possible. So let's consider the peak in the figure 12(b), where one can note the measured resonance frequency is

$$\log_{10}(f_{RM}) \approx 4 \Rightarrow f_{RM} \approx 10^4 = 10000\text{Hz},$$

which is smaller than theoretical value for this. But when taking the error of theoretical value into account, the measured resonance frequency corresponds the theoretical one.



(a) Theoretical attenuation (b) Measured attenuation

Figure 12: Theoretical and measured attenuation plots of LC-filter

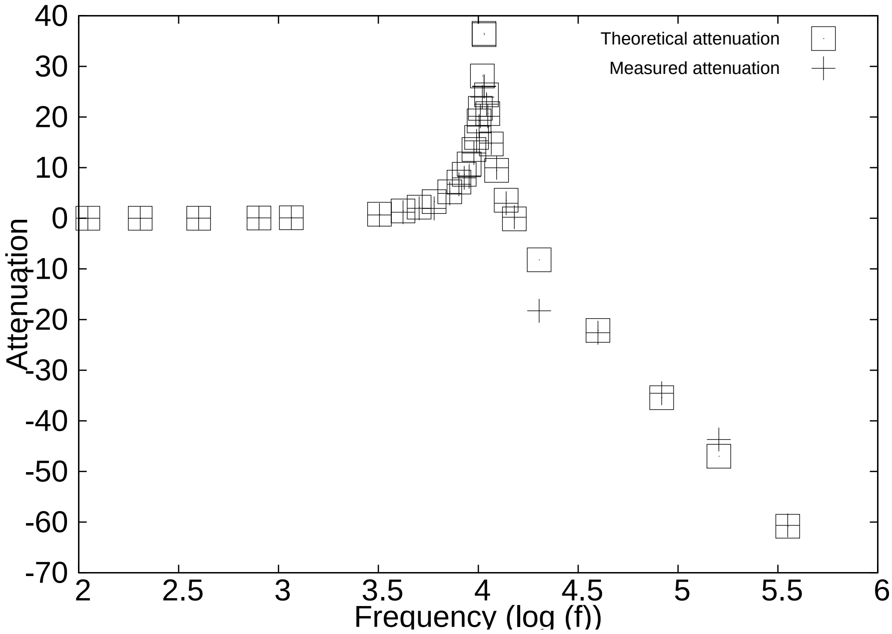


Figure 13: Measured and theoretical attenuations of RC-filter

4.2.2 Phase shift

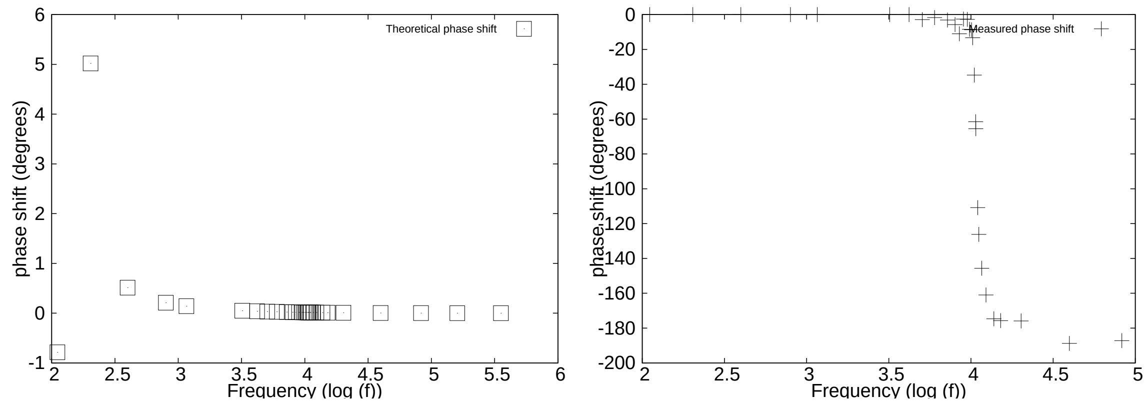
The theoretical phase shift for LC-filter can be calculated using the equation 9

$$\phi_T = \arctan\left(\frac{-\omega R_L C}{1-\omega^2 LC}\right) = \arctan\left(\frac{-2\pi f R_L C}{1-(2\pi f)^2 LC}\right) = \arctan\left(\frac{-360 f R_L C}{1-(360 f)^2 LC}\right)$$

and the measured phase shift is defined to be

$$\phi_M = 2\pi \Delta t f = -360 \Delta t f.$$

The theoretical and measured phase shift plottings are represented in the figures 14(a) and 14(b) and they are also plotted in the same figure 15.



(a) Theoretical phase shift (b) Measured phase shift

Figure 14: Theoretical and measured phase shift of LC-filter

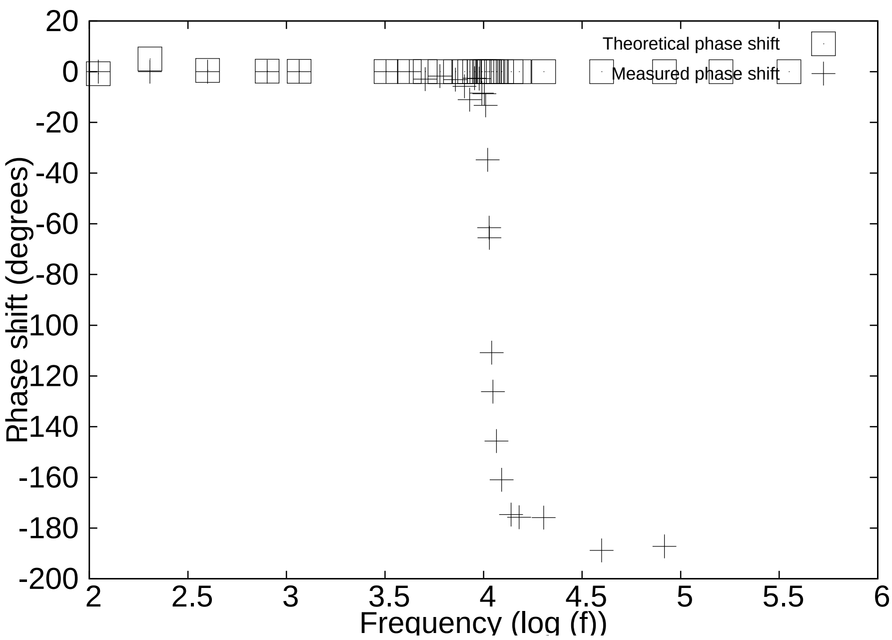


Figure 15: Measured and theoretical phase shift of LC-filter

4.3 RLC-filter

When measuring RLC-filter we used the same capacitor and inductor than in RC-filter and LC-filter measurements, but the resistor used in this case was different.

4.3.1 Attenuation

The theoretical attenuation can be calculated putting the equation 13 to the equation 2

$$\lambda_T = 20 \log_{10} \Big(\frac{1 - \omega^2 CL}{\sqrt{(1 - \omega^2 CL)^2 + (\omega CR)^2}} \Big),$$

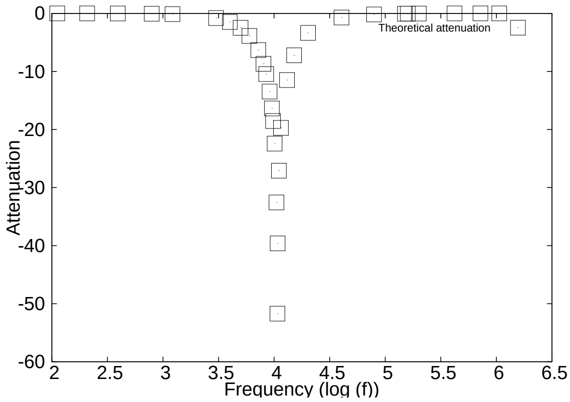
where the parameters of the components C, L and R are measured with multimeter and $\omega = 2\pi f$ is frequency. The measured attenuation is defined to be

$$\lambda_M = 20 \log_{10} \Big(\frac{V_{out}}{V_{in}} \Big)$$

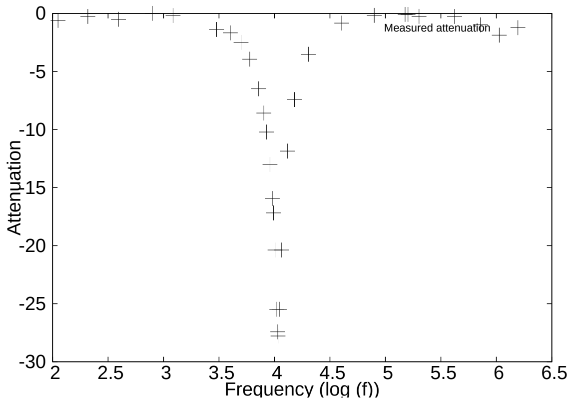
as like at the RC-filter and LC-filter case. Thus those input and output voltages are measured using the oscilloscope. Theoretical attenuation is plotted in figure 16(a) and measured attenuation is showed in figure 16(b). Both of the theoretical and measured attenuation plottings are represented in the same figure 19.

Theoretical value for resonance freqecuency can be calcuated with equation 14, which says $f_{RT} = \frac{1}{2\pi\sqrt{LC}} \approx 10686.6$Hz, which is exactly the same than the theoretical resonance frequency of LC-filter, because we used both of the cases the same capacitor and inductor. Thus also the error of the theoretical resonance frequency is same than in LC-filter case in the section 4.2.1 and we can express $f_{RT} = (11000 \pm 4000)$Hz. The measured value for this can be interrepted from the picture 16(b), where the resonance frequency occurs in the point, where is the peak. So the measured resonance frequency is $log_{10}(f_{RM}) = 4 \Rightarrow f_{RM} = 10000$Hz.

Q-value is defined as the equation 15, thus $Q = \frac{\omega_R L}{R}$. Calculating the theoretical value we need to use theoretical resonance frequency $\omega_{RT} = 2\pi f_{RT} = 67145.9\Omega$, which gives the theoretical Q-value $Q_T = \frac{\omega_{RT} L}{R} \approx 0.68$. The measured Q-value can be calculated with measured frequency $\omega_{RM} = 2\pi f_{RM} = 62831.85\Omega$, thus $Q_M = \frac{\omega_{RM} L}{R} \approx 0.64$.



(a) Theoretical attenuation



(b) Measured attenuation

Figure 16: Theoretical and measured attenuation of RLC-filter

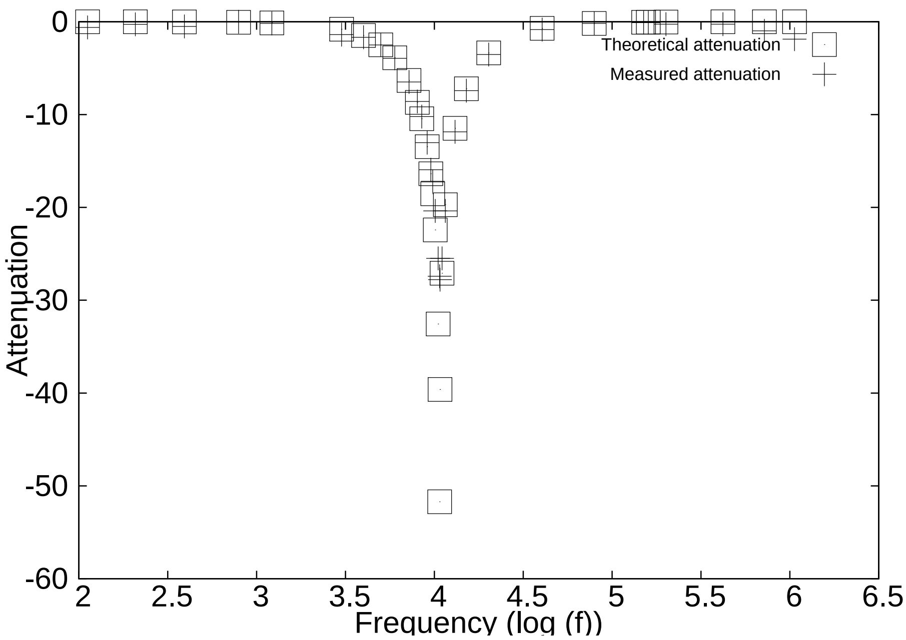


Figure 17: Measured and theoretical attenuation of RLC-filter

4.3.2 Phase shift

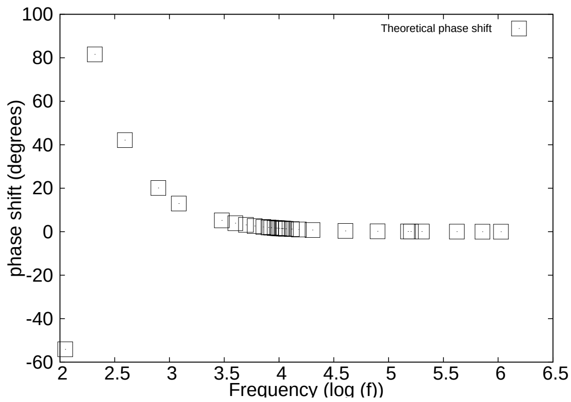
To obtain theoretical phase shift for RLC-filter one has to use the equation 12

$$\phi_T = \arctan\left(\frac{-\omega CR}{1-\omega^2 CL}\right) = \arctan\left(\frac{-360fCR}{1-(360f)^2 CL}\right)$$

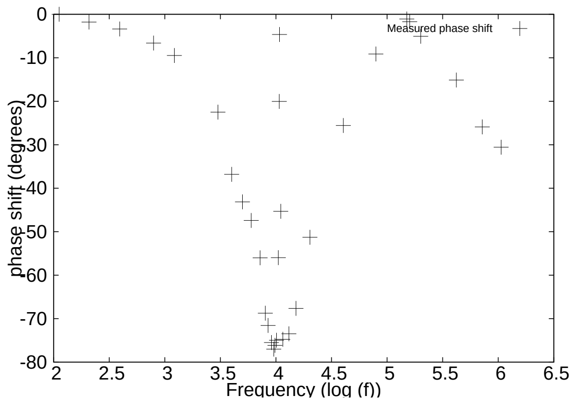
and it's plotted in figure 18(a) and the measured phase shift is

$$\phi_M = 2\pi\Delta t f = -360\Delta t f,$$

which is plotted in figure 18(b). The plotting of the measured phase shift seem to be like a triangle, allthough the theoretical plotting is a quite clean. The figure 19 shows both the theoretical and the measured the phase shift.



(a) Theoretical phase shift



(b) Measured phase shift

Figure 18: Theoretical and measured phase shift of RLC-filter

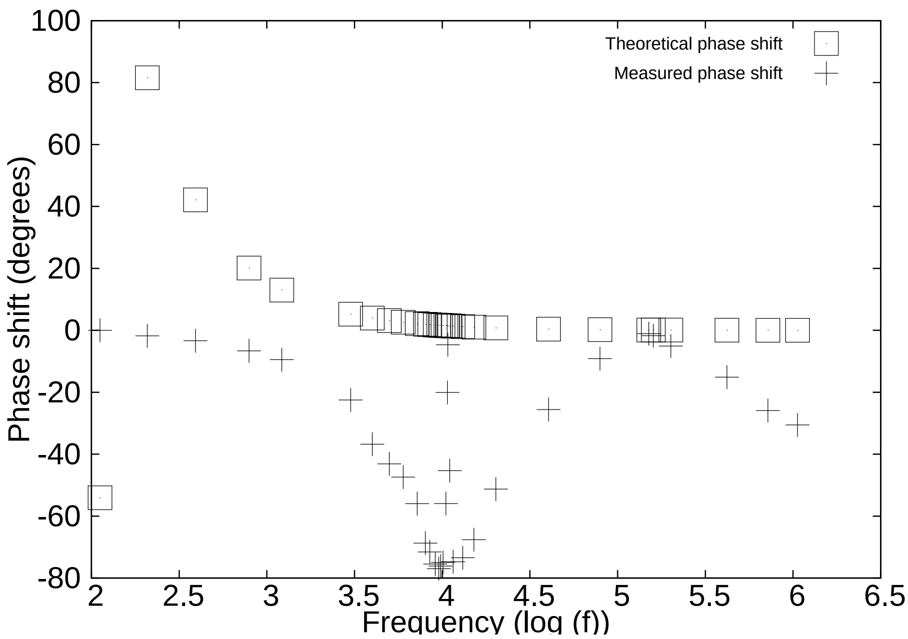


Figure 19: Measured and theoretical phase shift of RLC-filter

5 Conclusions

Let's first consider the results of RC-filter. The measured attenuation of RC-filter seemed to follow the theoretical attenuation in low frequencis as shown in the fiture 9, but in high frequencies the measured plottings don't decrease as fast as the theoretical plottings. It means the output voltage decreasing becomes slower. The measured cutoff frequency was $f_{cM} \approx 1003$Hz, which was smaller than theoretical $f_{cM} \approx 1075.80$Hz. Maybe those differencies between theoretical and measured values can be due to the resistore heating, when power is on. It may create the lower resistance affecting higher attenuation. The measured phase shift plotting was more near zero than the theoretical as shown in figure 11, which means there may have been something disturbance.

Next we consider the LC-filter results. Both of the theoretical and measured attenuation plottings in figure 13 seem to first be constant and then there is a peak, after which the attenuation begins to decrease. But the peak in theoretical plotting is a bit higher than measured one. After peak the measured plots seem almost follow the theoretical plots. Now one can note there is a difference to RC-filter, where the measured attenuation didn't decrease as fast as the theoretical one after. The difference between RC and LC-filter can be due to the fact that inner resistance of inductor is very small, maybe it doesn't heat so much as the larger resistance of the RC-filter. I calculated also theoretical $f_{RT} \approx 10686.6$Hz and measured $f_{RM} \approx 10000$Hz resonance frequency for LC-filter and one can note the theoretical is about 7% larger than measured one. Maybe the coil affect some magnetic fiel creating inductance in the circle and disturbs the resonance feature.

The phase shift in figure 15 of LC-filter almost followed the theoretical one in low frequencies but after resonance the theoretical plotting stagnated almost at zero, although the measured value was lowering getting negative values. Maybe there was generated some additional resistance or features of capacitor was transformed, which the theory doesn't notice.

Last one filter was the RLC-filter. I made also for this filter attenuation and phase shift plottings. The theoretical and measured attenuation plottings in figure 19 were very same alike with every frequency values, buth at the resonance the measured attenuation peak wasn't as deep as the theoretical one. Both of the theoretical and measured resonance frequency values was exactly the same than in LC-filter. Actual the theoretical resonance frequency was defined by using the same equation with same parameter values and measured resonance was defined from the attenuation figures finding the peak location. For RLC-filter I defined also the Q-value, which describes the quality of the circuit. Theoretical Q was $Q_T \approx 0.68$ and measured one was a bit smaller $Q_M \approx 0.64$, which means the peak of measured plotting should be wider and shorter than theoretical one, which is clearly shown in the figure 19. The phase shift of this filter was very different from the theoretical.

6 Attachments

1. Measurement log
2. gnuplot logs
3. Measurement result tables (used with gnuplot)

References

[1] B131 universal lcr meter. Manual on the internet: http://www.darlas.gr/comersus/store/catalog/pf/datasheets/52258.pdf.

[2] T. Denton. *Automobile electrical and electronic systems*. Butterworth-Heinemann, 4. edition, 2004. ISBN 9780750662192.

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