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Work performed: 1.11.2010

Fyss300/1 Impedance measurements

Supervisor:

Abstract

This study represents the calculated and measured impedance of unknown resistor in Wheatstone and unknown inductor in Maxwell bridge. We tried to balance both of the bridges by modifying the Helipot resistors until the voltage difference between two midpoints of the bridge was near enough zero. With Maxwell bridge we measured the inductor resistance and inductance and using them defined the impedance. Then we calculated the impedance of inductor by using the values of known Helipot resistors and capacitor in the circle. The calculated impedances seemed to be larger than measured ones in every measurement. Also with the Wheatstone bridge the calculated impedance varies from measured impedance. In both Maxwell and Wheatstone bridge the difference of measured and calculated impedance may be due to the facts that the voltage difference between midpoints wasn't exactly zero and the power supply can cause the heating of the resistors modifying the resistances.

Contents

1 Introduction

Impedance is a feature of circuit and it defines the relation of voltage and current.[1, p. 1191] Its unit is ohm as like resistance, but it can be discribed by complex number including real and imaginary part. That imaginary part comes from capasitance of a capacitor or inductance of a coil.

With the electrical components called by ”bridges” one can measure an accurate value for impedance of a some component in the bridge. We used two type of bridge: Wheatstone bridge and Maxwell bridge. The first one is more simple including only DC source and resistors. The another one is more complex including AC source, resistors, capacitor and coil.

2 Theoretical background

2.1 Impedance

Impedance can be defined separately for resistance, capasitance and inductance

$$\begin{aligned} Z_R &= R \\ Z_C &= \frac{-i}{\omega C} \\ Z_L &= i\omega L, \end{aligned} \tag{1}$$

where ω is angular frequency $2\pi f$ and i an imaginary unit.

If impedance of circuit is a complex number including resistance R and reactance X, it can be expressed as a function

$$Z = R + iX, \tag{2}$$

where the real part is R and imaginary part is X. In this kind of situation we have a complex equation of the form $Z = a + ib$, when the magnitude of the impedance can be expressed as

$$|Z| = \sqrt{a^2 + b^2}. \tag{3}$$

When we have serial impedance, the total impedance is

$$Z_{tot} = \sum_n Z_n. \tag{4}$$

In proportion in parallel case the total impedance is

$$\frac{1}{Z_{tot}} = \sum_n \frac{1}{Z_n}. \tag{5}$$

2.2 Wheatstone bridge

Wheatstone bridge includes DC voltage supply and four resistors as shown in figure 1. Because there is not capasitors or coils, there is no need to use imaginary parts in this case.

We know the resistances R_1, R_2 and R_3, but the R_x is unknown. On the left side of the circuit is current I_a and on in right side I_b. If the potential different between $U(P_1)$ and $U(P_2)$ is zero, the bridge is "balanced". Then the potential should be same in a point P_1 and P_2, thus we can conclude

$$I_a R_1 = I_b R_2$$
$$I_a = I_b \frac{R_2}{R_1}. \tag{6}$$

We know the voltage drop through the resistances R_1 and R_3 is same as voltage drop through the resistances R_2 and R_x, so we can write

$$I_a R_3 + I_a R_1 = I_b R_x + I_b R_2, \tag{7}$$

where we put the current given by equation 6

$$I_b \frac{R_2}{R_1} R_3 + I_b \frac{R_2}{R_1} R_1 = I_b R_x + I_b R_2$$
$$R_x = \frac{R_2 R_3}{R_1}. \tag{8}$$

Using the equation 8 we can calculate the unknown resistance R_x,

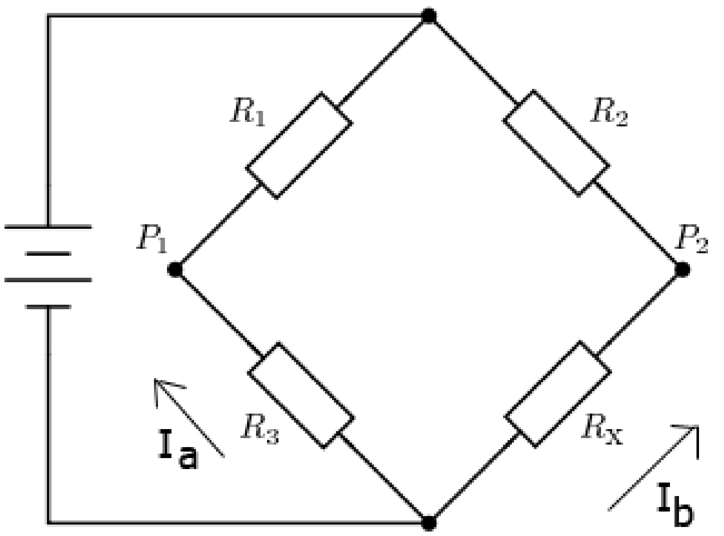

Figure 1: Wheatstone bridge

2.3 Maxwell bridge

Maxwell bridge is represented in figure 2, where it's set with alternating current source. Structure of Maxwell bridge includes three known resistors R_1, R_2 and R_3 and one unknown resistor R_x. Additionally there is capacitor and coil. We know the capacitance C_1, but the inductance L_x is not known.

When handling Maxwell bridge, we have to use complex numbers including the imaginary components represented in section 2.1.

When Maxwell bridge is balanced, the voltages U_{P_1} and U_{P_2} are the same. That's why we can write

$$I_d \frac{-i}{\omega C} = I_c R_1 = I_b R_2. \tag{9}$$

As shown in figure 2 the current I_a is divided to two parts I_c and I_d. So using equation 9 we can express current I_a

$$I_a = I_c + I_d = \frac{I_b R_2}{R_1} - \frac{I_b R_2 \omega C}{i}. \tag{10}$$

Now let's consider the fact potential difference between $U(P1)$ and $U(P_2)$ is zero

$$\begin{aligned} I_a R_3 - (I_b R_x + I_b i\omega L) = 0 \\ R_x = \frac{I_a R_3}{I_b} - i\omega L. \end{aligned} \tag{11}$$

Thus we got an equation for unknown resistance R_x. By making an assigment with equation 10 for equation 11 we obtain

$$R_x = \frac{R_2 R_3}{R_1} - \frac{R_2 R_3 \omega C}{i} - i\omega L. \tag{12}$$

Because the value of resistance should be real, we can conclude

$$R_x = \frac{R_2 R_3}{R_1}. \tag{13}$$

And the imaginary part of the equation 12 should be zero, when

$$\begin{aligned} -\frac{R_2 R_3 \omega C}{i} - i\omega L = 0 \\ L = R_2 R_3 C. \end{aligned} \tag{14}$$

Considering a balanced AC bridge we can calculate an unknown impedance using an equation

$$\begin{aligned} Z_x Z_1 = Z_2 Z_3 \\ \Rightarrow Z_x = \frac{Z_2 Z_3}{Z_1}, \end{aligned} \tag{15}$$

which is valid also for Maxwell bridge.

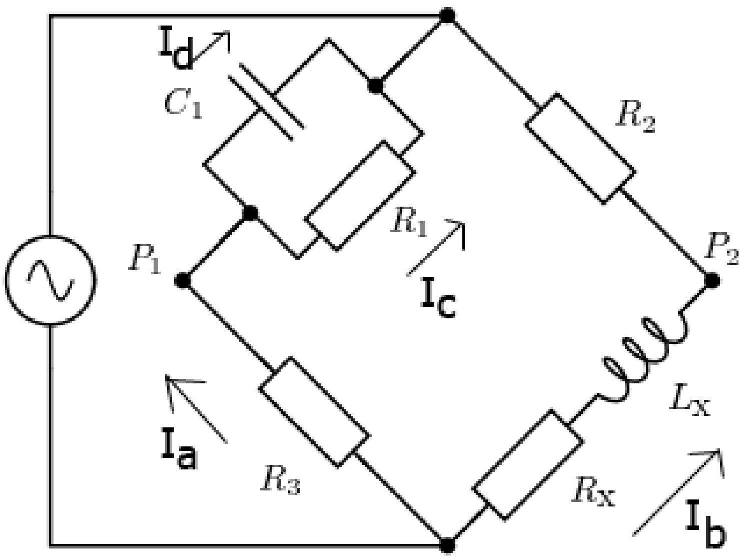

Figure 2: Maxwell bridge

2.4 propagation of uncertainty

When handling an error it's useful to use propagation of uncertainty method, which says

$$\delta f = \sqrt{\sum_i (\frac{\partial f}{\partial x_i} \delta x_i)^2}, \tag{16}$$

where δf is an error of f and x_i $(i = 1, 2..)$ are the variables in function of f.

3 Experimental methods

3.1 Maxwell bridge

We started the measurements with the more complex circuit - Maxwell bridge. Before measurements the supervisor had set up the coupling for Maxwell bridge presented in picture 3. The bridge was built with the connection board (Pimboard 3), which was connected with Maxcom MX-9300 power supply. The Helipots resistors are R_1, R_2 and R_3 in the picture. There were setted also a capacitor in parallel with resistor R_1. The inductor had a resistance R_x, so we didn't need a separate fourth resistor. The Kenwood CS-4125 oscilloscope was connected between the points P_1 and P_2 and it aimed to measure the potential difference between those points using two channels.

First we connected only the power supply and oscilloscope together. We tested the oscilloscope configuring the buttons using power supply frequency 1 kHz, until the oscilloscope seemed to represent signal image soundly. Then we connected the actual measurement coupling.

When measuring we modified the resitances of the Helipot resistors, until the voltage difference image on the oscilloscope screen seemed to be minimum and the phases of signals were the same. We used the same sensitivities with both of the channels. The voltage difference was defined by calculating the height of image from downward peak to upward peak. After every measurement like this we measured the resistances with Finest multimeter. We repeated this five times with different resistance combinations.

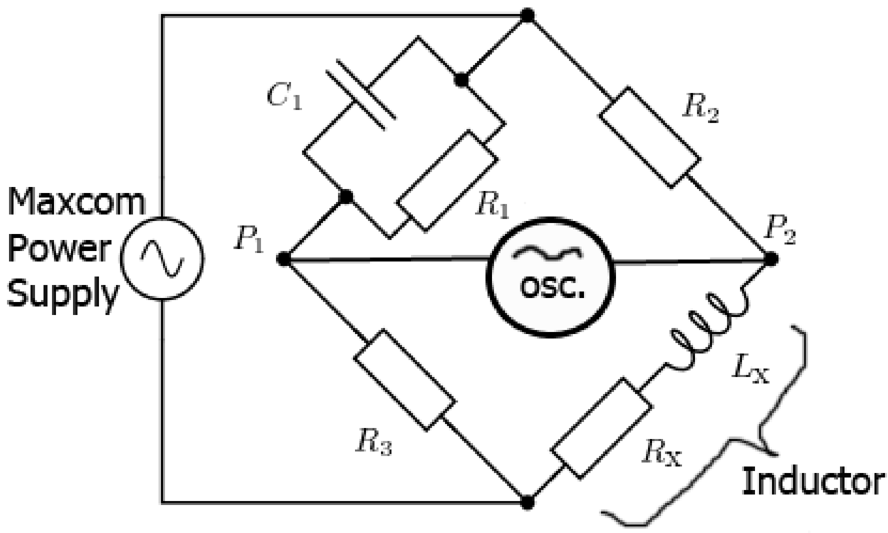

Figure 3: Set up for Maxwell bridge

3.2 Wheastone bridge

In the second measurement we used the Wheatstone bridge coupling presented in figure 4 using GW Laboratory DC Power supply (MODEL-GPS-3030). But instead of two separate resistors R_1 and R_2 we used one Helipot resistor connected to the circle to give two different resistances R_1 and R_2. The R_3 is also Helipost resistor and R_x is non-variable "unknown" resistor. The voltage difference between the points P_1 and P_2 was measured with Finest multimeter.

We modified the Helipot resistors until the voltage difference was small enough and then we measured the resisntaces of the Helipots with Finest multimeter. With this coupling system we performed only three measurements.

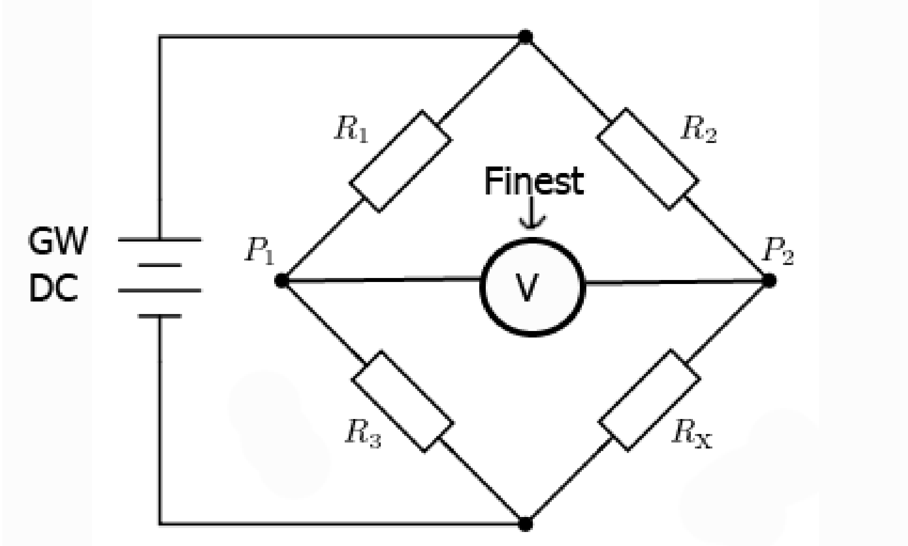

Figure 4: Set up for Maxwell bridge bridge

4 Results

For the most complicated calculations I used Wolfram Mathematica 6.0 and the calculation logs are shown in the attachment 2. I estimated the errors of the measurement equipments (Finest 203 and 387) to be 1% + 3 digits for resistance and inductance. For capacitance I estimated the error 2% + 2 digits.

4.1 Maxwell bridge

The table 1 shows the measured values for voltage difference V, Helipot resistances R_1, R_2 and R_3, inductor's resistance R_{xM} and inductor's inductance L_{xM} measured with multimeter. There are presented also both the calculated resistance R_{xC} and calculated inductance L_{xC} of the inductor.

Table 1: Maxwell bridge

f (kHz)	V (mV)	R_1 (kΩ)	R_2 (kΩ)	R_3 (Ω)	R_{xC} (Ω)	R_{xM} (Ω)	L_{xC} (mH)	L_{xM} (mH)
1	6	1.308	6.95	5.0	26.60	19.50	11.68	10.05
1	3	1.6	7.18	5.0	22.44	19.50	12.06	10.05
1	5	7.5	6.28	5.5	4.61	19.50	11.61	10.05
10	4	7.5	6.03	5.5	4.42	19.50	11.14	10.05
10	6	1.6	6.03	5.5	20.73	19.50	11.14	10.05

4.1.1 Calculated impedance

The calculated values for resistance, inductance, impedance and errors of them are all listed in the table 2.

Inductor's resistance and its' error

Using equation 13 we can calculate the resistance R_{xC} of inductor

$$R_{xC} = \frac{R_2 R_3}{R_1}. \tag{17}$$

For example with the first measured values of Helipot resistors ($R_1 = 1.308\text{k}\Omega$, $R_2 = 6.95\text{k}\Omega$, $R_3 = 5.0\Omega$) one can express the inductor's resistance

$$R_{xC} = \frac{6.95\text{k}\Omega \cdot 5.0\Omega}{1.308\text{k}\Omega} \approx 26.6\Omega.$$

Now it's needed to define the error for resistance. Let's consider the equation 16 using the resistance equation 17, thus it gives

$$\delta R_{xC} = \sqrt{(\frac{\partial R_{xC}}{\partial R_1}\delta R_1)^2 + (\frac{\partial R_{xC}}{\partial R_2}\delta R_2)^2 + (\frac{\partial R_{xC}}{\partial R_3}\delta R_3)^2},$$

where the partial derivates are

$$\frac{\partial R_{xC}}{\partial R_1} = \frac{-R_2 R_3}{R_1^2}$$
$$\frac{\partial R_{xC}}{\partial R_2} = \frac{R_3}{R_1}$$
$$\frac{\partial R_{xC}}{\partial R_3} = \frac{R_2}{R_1},$$

which we use to express the error

$$\delta R_{xC} = \sqrt{(\frac{-R_2 R_3 \delta R_1}{R_1^2})^2 + (\frac{R_3 \delta R_2}{R_1})^2 + (\frac{R_2 \delta R_3}{R_1})^2}. \tag{18}$$

For example at the first measurement the errors of Helipot resistances are $\delta R_1 = (0.01 * 1308 + 3) = 16.08\Omega$, $\delta R_2 = (0.01 * 6950 + 3)\Omega = 72.5\Omega$ and $\delta R_3 = (0.01 * 5 + 0.3)\Omega = 0.35\Omega$. Now we can calculate the inductor resistance error using equation 18:

$$\delta R_{xC} = \sqrt{(\frac{-6950 \cdot 5 \cdot 16.08}{1308^2})^2 + (\frac{5 \cdot 72.5}{1308})^2 + (\frac{6950 \cdot 0.35}{1308})^2}\ \Omega \tag{19}$$
$$\approx 1.9084\Omega.$$

Thus we have calculated the resistance and its' error for first case and we got $R_{xC} = (27 \pm 2)\Omega$. This calculation have been made for every measurement values and are shown in table 2.

Inductor's inductance and its' error

Then let's calculate the inductance using the equation 14

$$L_{xC} = R_2 R_3 C, \tag{20}$$

where C is capacitance. Then using the values $R_2 = 6.95\text{k}\Omega$ and $R_3 = 5.0\Omega$ from the table 1 and the measured capacitance $C = 336.4 \cdot 10^{-9}\text{F}$ we get

$$L_{xC} = 6.95\text{k}\Omega \cdot 5.0\Omega \cdot 336.4 \cdot 10^{-9}\text{F} \approx 11.68\text{mF}. \tag{21}$$

The inductor resistance and inductance values for the other measurements were calculated in the same way.

Next we figure out the inductance errors using again the propagation of error method:

$$\delta L_{xC} = \sqrt{(\frac{\partial L_{xC}}{\partial R_2}\delta R_2)^2 + (\frac{\partial L_{xC}}{\partial R_3}\delta R_3)^2 + (\frac{\partial L_{xC}}{\partial C}\delta C)^2}, \tag{22}$$

where the partial derivatives are

$$\begin{aligned} \frac{\partial L_{xC}}{\partial R_2} &= R_3 C \\ \frac{\partial L_{xC}}{\partial R_3} &= R_2 C \\ \frac{\partial L_{xC}}{\partial C} &= R_2 R_3. \end{aligned}$$

Substituting those partial derivates to equation of inductance error we obtain

$$\delta L_{xC} = \sqrt{(R_3 C \delta R_2)^2 + (R_2 C \delta R_3)^2 + (R_2 R_3 \delta C)^2}. \tag{23}$$

Let's then calculate the error for the first measurement using the same Helipot resistance errors than in equation 19. The capacitor value $C = 336.4 \cdot 10^{-9}\text{F}$ and error is $\delta C = 0.02 \cdot C + 0.2 \cdot 10^{-9}\text{F} = 6.728 \cdot 10^{-9}\text{F}$. Then we make the substitution

$$\begin{aligned} \delta L_{xC} = \Big(& (5.0 \cdot 336.4 \cdot 10^{-9} \cdot 72.5)^2 + (6950 \cdot 336.4 \cdot 10^{-9} \cdot 0.35)^2 \\ & + (6950 \cdot 5.0 \cdot 6.728 \cdot 10^{-9})^2 \Big)^{\frac{1}{2}} \text{ H} \\ & \approx 0.0008616\text{H}. \end{aligned} \tag{24}$$

Now we have obtained the inductance with its' error for first measurement $L_{xC} = (0.0117 \pm 0.0009)\text{H}$. The inductance calculations for all of the measurements are shown in the attachment 2 and the results are listed in the table 2.

Inductor's impedance and its' error

Now we have the values for resistance and inductance of inductor, thus let's consider the impedance. Because we can handle the resistance and inductance of the inductor as the components connected in the series, we can use the impedance equation 4

$$Z_{xC} = Z_{RC} + Z_{LC}.$$

Applying the equation 1 we get $Z_{RC} = R_{xC}$ and $Z_{LC} = i\omega L_{xC}$, thus the total impedance of inductor is

$$Z_{xC} = R_{xC} + i\omega L_{xC}, \tag{25}$$

where i is an imaginary component. Then we transform the equation to give magnitude of impedance using equation 3

$$|Z_{xC}| = \sqrt{(R_{xC})^2 + (\omega L_{xC})^2}. \tag{26}$$

Now we calculate the impedance for the first resistance and inductance values on the table 2

$$\begin{aligned} |Z_{xC}| &= \sqrt{(26)^2 + (2\pi \cdot 1000 \cdot 0.0117)^2}\ \Omega \\ &\approx 77.9756\Omega. \end{aligned} \tag{27}$$

Now let's figure out the error for impedance

$$\delta|Z_{xC}| = \sqrt{(\frac{\partial|Z_{xC}|}{\partial R_{xC}}\delta R_{xC})^2 + (\frac{\partial|Z_{xC}|}{\partial L_{xC}}\delta L_{xC})^2}, \tag{28}$$

where the partial derivates are

$$\begin{aligned} \frac{\partial|Z_{xC}|}{\partial R_{xC}} &= \frac{1}{2}\frac{2R_{xC}}{\sqrt{(R_{xC})^2 + (\omega L_{xC})^2}} = \frac{R_{xC}}{\sqrt{(R_{xC})^2 + (\omega L_{xC})^2}} \\ \frac{\partial|Z_{xC}|}{\partial L_{xC}} &= \frac{1}{2}\frac{2\omega L_{xC}}{\sqrt{(R_{xC})^2 + (\omega L_{xC})^2}} = \frac{\omega L_{xC}}{\sqrt{(R_{xC})^2 + (\omega L_{xC})^2}}. \end{aligned}$$

Substituting those deriveates to equation 28 we obtain

$$\delta|Z_{xC}| = \sqrt{\frac{(R_{xC}\delta R_{xC})^2 + (\omega L_{xC}\delta L_{xC})^2}{(R_{xC})^2 + (\omega L_{xC})^2}}. \tag{29}$$

Now we calculate the impedance error for the first measurement using the values in first line of table 2

$$\begin{aligned} \delta|Z_{xC}| &= \sqrt{\frac{(26 \cdot 2)^2 + (2\pi \cdot 1000 \cdot 0.0117 \cdot 0.0009)^2}{(26)^2 + (2\pi \cdot 1000 \cdot 0.0117)^2}}\ \Omega \\ &\approx 0.6669.\Omega \end{aligned}$$

Finally we got the value for impedance $|Z_{xC}| = (78.0 \pm 0.7)\Omega$. Also all the calculated values for impedance and its errors are shown in table 2

Table 2: Calculated resistances, inductances and impedances of inductor

f (kHz)	R_{xC} (Ω)	δR_{xC} (Ω)	L_{xC} (H)	δL_{xC} (H)	$\lvert Z_{xC}\rvert$ (Ω)	$\delta\lvert Z_{xC}\rvert$ (Ω)
1	26	2	0.0117	0.0009	78.0	0.7
1	22	2	0.0121	0.0009	79.1	0.6
1	4.6	0.4	0.0116	0.0008	73.03	0.03
10	4.4	0.3	0.0111	0.0008	697.447	0.003
10	20.7	1.4	0.0111	0.0008	697.74	0.05

4.1.2 Measured impedance

During the measurements we have measured the resistance $R_{xM} = 19.5\Omega$ and inductance $L_{xM} = 10.05\text{mH}$ of inductor using a multimeter. The error of resistance is the error of multimeter shown in the begin of section 4. Thus $\delta R_{xM} = (0.01 * R_{xM} + 0.03)\Omega = 0.225\Omega$. Now we can express the measured resistance and its error:

$$R_{xM} = (19.5 \pm 0.03)\Omega. \tag{30}$$

In proportional we can handle the inductance of inductor. The error for this can also be defined as an error of multimeter: $\delta L_{xM} = (0.01 * L_{xM} + 0.00003)\text{H} = 0.0001305\text{H}$. Now the inductance and its error can be wrote

$$L_{xM} = (0.01005 \pm 0.00015)\Omega. \tag{31}$$

The inductor includes resistance and inductance and we can imagine they are connected in series thus considering the equation 4 the impedance is

$$Z_{xM} = Z_{RxM} + Z_{LxM} = R_{xM} + i\omega L_{xM},$$

where is both a complex and an imaginar component, when we need to use the equation 3. Thus with the frequency $\omega = 2\pi \cdot 1000\text{Hz}$ the magnitude of impedance is

$$\begin{aligned}|Z_{xM}| &= \sqrt{(R_{xM})^2 + (\omega L_{xM})^2}\\ &= \sqrt{19.5^2 + (2\pi * 1000 \cdot 0.01005)^2}\ \Omega\\ &\approx 66.088\Omega.\end{aligned}$$

And in case of frequency $\omega = 2\pi \cdot 10000\text{Hz}$ we can write

$$\begin{aligned}|Z_{xM}| &= \sqrt{19.5^2 + (2\pi \cdot 10000 \cdot 0.01005)^2}\ \Omega\\ &\approx 631.761\Omega.\end{aligned}$$

Let's then define the error for this using the equation 16

$$\delta|Z_{xM}| = \sqrt{(\frac{\partial|Z_{xM}|}{\partial R_{xM}}\delta R_{xM})^2 + (\frac{\partial|Z_{xM}|}{\partial L_{xM}}\delta L_{xM})^2},$$

where the partial derivates are

$$\frac{\partial|Z_{xM}|}{\partial R_{xM}} = \frac{R_{xM}}{\sqrt{(R_{xM})^2 + (\omega L_{xM})^2}}$$

and

$$\frac{\partial|Z_{xM}|}{\partial L_{xM}} = \frac{\omega L_{xM}}{\sqrt{(R_{xM})^2 + (\omega L_{xM})^2}}.$$

Using those partial derivates we can express

$$\delta|Z_{xM}| = \sqrt{\frac{(R_{xM}\delta R_{xM})^2 + (\omega L_{xM}\delta L_{xM})^2}{(R_{xM})^2 + (\omega L_{xM})^2}},$$

where $R_{xM} = 19.5\Omega$, $\delta R_{xM} = 0.225\Omega$, $L_{xM} = 10.05\text{mH}$ and $\delta L_{xM} = 0.0001305\Omega$ as shown earlier. In case of $\omega = 2\pi \cdot 1000\text{Hz}$ we get

$$\delta|Z_{xM}| \approx 0.0664\Omega \;\Rightarrow\; |Z_{xM}| = (66.09 \pm 0.07)\Omega \tag{32}$$

and when $\omega = 2\pi \cdot 10000\text{Hz}$ we get

$$\delta|Z_{xM}| \approx 0.0069\Omega \;\Rightarrow\; |Z_{xM}| = (631.761 \pm 0.007)\Omega. \tag{33}$$

4.2 Wheatstone bridge

The values for Wheatstone bridge measured during the measurement are shown in the table 3.

Table 3: Wheatstone bridge

V (mV)	R_1 (kΩ)	R_2 (kΩ)	R_3 (kΩ)	R_{xM} (kΩ)
0.3	6.59	3.41	1.91	0.988
0.2	8.62	0.96	6.46	0.710
$\approx$ 0.0	7.64	0.98	6.48	0.945

In Wheatstone birdge the impedance of resistor R_x is actually the same as the value of resistance $Z_x = R_x$.

4.2.1 Measured impedance

Now we can conclude the impedance for the first measurement is $Z_{xM} = R_{xM} = 988\Omega$.

For measured impedance values the error is same as the error of multimeter. Thus for the first measurement we get the error

$$\delta Z_{xM} = (0.01 * 988 + 3)\Omega = 12.88\Omega. \tag{34}$$

So the impedance is $Z_{xM} = (990 \pm 15)\Omega$. One should note that rounding an error, it can be made at the most 15 significant number accuracy. Thus any number between 10 and 15 can be rounded to be 15. The values for measured impedances with errors are shown in table 4.

4.2.2 Calculated impedance

We can also calculate the impedance using values R_1, R_2 and R_3 on the table 3 using the equation 8.

$$Z_{xC} = R_{xC} = \frac{R_2 R_3}{R_1}. \tag{35}$$

Let's now substitute the first measurement values to the equation above

$$R_{xC} = \frac{3410 \cdot 1910}{6590}\Omega \approx 988.3308\Omega. \tag{36}$$

The error is

$$\delta Z_{xC} = \sqrt{(\frac{\partial Z_{xC}}{\partial R_1}\delta R_1)^2 + (\frac{\partial Z_{xC}}{\partial R_2}\delta R_2)^2 + (\frac{\partial Z_{xC}}{\partial R_3}\delta R_3)^2}, \tag{37}$$

where partial derivates

$$\begin{aligned} \frac{\partial Z_{xC}}{\partial R_1} &= \frac{-R_2 R_3}{R_1^2} \\ \frac{\partial Z_{xC}}{\partial R_2} &= \frac{R_3}{R_1} \\ \frac{\partial Z_{xC}}{\partial R_3} &= \frac{R_2}{R_1} \end{aligned} \tag{38}$$

and substituting them we get

$$\delta Z_{xC} = \sqrt{(\frac{-R_2 R_3}{R_1^2}\delta R_1)^2 + (\frac{R_3}{R_1}\delta R_2)^2 + (\frac{R_2}{R_1}\delta R_3)^2}. \tag{39}$$

Let's again calculate the error for first case using the multimeter errors for Helipots $\delta R_1 = (0.01 * 6590 + 3)\Omega = 68.9\Omega$, $\delta R_2 = (0.01 * 3410 + 3)\Omega = 37.1\Omega$ and $\delta R_3 = (0.01 * 1910 + 3)\Omega = 22.1\Omega$

$$\delta Z_{xC} = \sqrt{(\frac{-3410 \cdot 1910}{6590^2}68.9)^2 + (\frac{1919}{6590}37.1)^2 + (\frac{3410}{6590}22.1)^2}\ \Omega \approx 18.7929\Omega.$$

I made the same kind of calculation also for all the other values and they are shown in attachment 2. The results are in the table 4.

Table 4: Calculated and measured impedance with errors

Z_{xM} (Ω)	δZ_{xM} (Ω)	Z_{xC} (Ω)	δZ_{xC} (Ω)
990	15	990	20
710	15	719	15
950	15	830	20

5 Conclusions

Let's first consider the results for Maxwell bridge case. When the frequency was $f = 1000$Hz, the measured impedance was $Z_{xM} = (66.09 \pm 0.07)\Omega$ and the calculated was between $Z_{xC} = 73.03\Omega$ and $Z_{xC} = 79.1\Omega$. For the frequency $f = 10000$Hz the measured impedance was $Z_{xM} = (631.761 \pm 0.007)\Omega$ and calculated was between $Z_{xC} = (697.447 \pm 0.003)\Omega$ and $Z_{xC} = (697.74 \pm 0.05)\Omega$. The calculated value isn't a constant, but it seem to change.

Anyway in both of frequencies the calculated impedance Z_{xC} is higher than measured impedance Z_{xM}. This can be due to the fact that the voltage difference between points P_1 and P_2 in the bridge was not zero, but few millivolts, which means the bridge was not exactly balanced. We couldn't obtain the proper resistance value for balanced condition thus using these "wrong" resistance values we got "wrong" result for impedance.

The another reason for this impedance difference may be the thermal features of the Helipot resistors. The resistors were modified during the power supply was turned on, which may heat the resistor. And when the resistors were measured with multimeter they were separated from the circuit, which may inflict temperature dropping. Thus we might have measured the resistance of "cold" resistor, which may differs from the value when it was connected. So we used these "wrong" resistance values when calculated the impedance.

Let's next consider the Wheatstone bridge. The calculated Z_{xC} and measured values Z_{xM} for impedances are shown in the table 4. The first values was about the same $Z_{xM} = (990 \pm 15)\Omega$ and $Z_{xC} = (990 \pm 20)\Omega$. In the second measurement the calculated value is a bit larger than the measured ($Z_{xM} = (710 \pm 15)\Omega$, $Z_{xC} = (719 \pm 15)\Omega$) and in the last measurement the calculated impedance is notably smaller than the measured one (($Z_{xM} = (950 \pm 15)\Omega$, $Z_{xC} = (830 \pm 20)\Omega$)). One can note at least there is not systematic error.

Maybe also in this situation the reason is the non-zero voltage difference and the heating of the resistors, which skews the results when calculating.

6 Attachments

1 Measurement log
2 Wolfram Mathematica logs

References

[1] Young and Freedman. *University Physics with Modern Physics*. Pearson, 11. edition, 2004. ISBN 0-321-20469-7.